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MODELING AND SOLVING EQUATIONS OF THE WAVE PROCESS USING THE RECURRENT OPERATOR METHOD

P. M. Pirniyazova*

Almaty Technological University, Almaty, Kazakhstan

** E-mail for contacts: pirniyazova1974@gmail.com*

This article is a review, which provides a solution to the one-dimensional and three-dimensional problem of the wave process. The equation in question is a hyperbolic equation that can be derived only for the one-dimensional case and describes the process of river flow, matter transfer, diffusive mass transfer, and flow through a porous medium. The solution of the equation is sought in the form of integro-differential operators, represented as a special series with constant coefficients determined from the recurrence relation. With this approach, general solutions are expressed in terms of arbitrary functions and are not related to solving another equation. The resulting form of general solutions makes it possible to apply the method of initial functions to solve boundary value problems, since arbitrary analytical functions included in general solutions can be expressed in terms of initial functions specified by the problem condition. From these formulas of general solutions, it is easy to find all particular solutions in various classes of analytical functions. Next, the solution of a special case of the Lame equation describing the wave process is considered. The solution of a special case of the Lame equation is also sought in the form of a series, using the recurrent operator method. The algorithm for solving the three-dimensional problem of the wave process is similar to that for the one-dimensional problem. This approach is very effective for constructing solutions to the equations of the theory of thermal conductivity and the theory of elasticity. The results obtained are illustrated graphically and shown in the tables.

Keywords: *wave process, special case of the Lame equation, recurrent operator method.*

INTRODUCTION

In this article, we will consider the general solution of the differential equation of the wave process, which determines the physical processes of interest to us, and a number of other physical processes are determined by similar equations, among them potential flow, diffusive mass transfer [1, 2], flow through a porous medium and some fully developed flows in channels. Let us consider the process of river flow and transfer of substance by a one-dimensional differential equation, a hyperbolic type deduced only for the one-dimensional case [1], based on the theory of collision of solid particles, is also only some approximation to an adequate description of turbulent motion.

$$\frac{\partial^2 q}{\partial t^2}+a\frac{\partial q}{\partial t}+D_x\frac{\partial^2 q}{\partial t^2}-U_x\frac{\partial q}{\partial x}+\gamma q=f\left(x,t\right) \qquad (1)$$

where $\partial_x^2=\frac{\partial^k}{\partial s^k}$; $a_1=-\frac{1}{D_x}$; $a_2=-\frac{U_x}{D_x}$; $a_3=-\frac{a}{D_x}$; $a_4=-\frac{\gamma}{D_x}$; $f=\frac{f^*}{D_x}$; $q(x,t)$ – diffusion intensity, U_x – the rate of propagation of the substance along the axis , a – the porosity coefficient of the liquid, D_x – the diffusion coefficient along the axis ox , γ – the rate of destruction of the substance, $f\left(x,t\right)$ – the source member.

Let's write this equation in the operator form

$$\left(\partial_x^2+a_1\partial_t^2+a_2\partial_x+a_3\partial_t+a_4\right)q\left(x,t\right)=f\left(x,t\right) \qquad (2)$$

In accordance with the recurrent operator method [3, 4], we are looking for a solution to the homogeneous equation of the equation at $f=0$, in the form of a series

$$q_r\left(x,g_r\right)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}Q_{i,j}\frac{x^{(i+j+r)!}}{\left(i+j+r\right)!}\frac{\partial^j}{\partial t^j}g_r\left(t\right);\ r=0,1 \qquad (3)$$

where $Q_{i,j}$ – constant coefficients of the series determined from the condition for $i<0$, or $j<0$, then $Q_{i,j}=0$ and when $i=0,j=0$; then $Q_{i,j}=1$.

Substituting the series (3) into equation (1) we obtain

$$\begin{gathered}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}Q_{i,j}\frac{x^{i-2+j+r!}}{\left(i-2+j+r\right)!}\frac{\partial^j}{\partial t^j}g_r\left(t\right)+\\+a_1\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}Q_{i,j}\frac{x^{i+j+r!}}{\left(i+j+r\right)!}\frac{\partial^{j+2}}{\partial t^{j+2}}g_r\left(t\right)+\\+a_2\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}Q_{i,j}\frac{x^{i-1+j+r!}}{\left(i-1+j+r\right)!}\frac{\partial^j}{\partial t^j}g_r\left(t\right)+\\+a_3\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}Q_{i,j}\frac{x^{i+j+r!}}{\left(i+j+r\right)!}\frac{\partial^{j+1}}{\partial t^{j+1}}g_r\left(t\right)+\\+a_4\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}Q_{i,j}\frac{x^{i+j+r!}}{\left(i+j+r\right)!}\frac{\partial^j}{\partial t^j}g_r\left(t\right)=0\end{gathered} \qquad (4)$$

Making the indices offset in the second sum $j\to j-2$ in the third $i\to i-1, j\to j-1$, in the fourth amount third $i\to i-1$, in the fifth sum of the series third

$i \to i-2$, and taking out the total multiplier $\frac{x^{i-2+j+r!}}{(i-2+j+r)!}\frac{\partial^j}{\partial t^j}g_r(t)$ we get the parentheses

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[Q_{i,j}+a_1Q_{i,j-2}+a_2Q_{i-1,j}+a_3Q_{i-1,j-1}+a_4Q_{i-2,j}\right]\cdot \cdot\frac{x^{i-2+j+r!}}{(i-2+j+r)!}\frac{\partial^j}{\partial t^j}g_r(t)=0 \tag{5}$$

Equating the expressions in square brackets to zero, we obtain the following recurrence relation

$$Q_{i,j}=-a_1Q_{i,j-2}-a_2Q_{i-1,j}-a_3Q_{i-1,j-1}-a_4Q_{i-2,j} \tag{6}$$

under the conditions $i<0$, or $j<0$, then $Q_{i,j}=0$, when $i=0, j=0$; then $Q_{i,j}=1$.

Let's write out the first few expressions for the coefficients $Q_{i,j}$

$Q_{0,0}=1$; $Q_{0,1}=0$; $Q_{0,2}=-a_1$; $Q_{0,3}=0$; $Q_{0,4}=a_1^2$;
$Q_{1,0}=-a_2$; $Q_{1,1}=-a_3$; $Q_{1,2}=2a_1a_2$; $Q_{1,3}=2a_1a_3$;
$Q_{2,0}=a_2^2-a_4$; $Q_{2,1}=2a_1a_3$; $Q_{2,2}=-3a_1a_2^2+2a_1a_4+a_3^2$;
$Q_{3,0}=-2a_2^2a_3+a_2a_4$; $Q_{3,1}=-3a_2^2a_3+2a_1a_4+a_3^2$;
$Q_{4,0}=-2a_2^3a_3-2a_2^2a_4+a_4^2$.

By the method of full induction, it can be shown that the solution of the recurrent equation (6) has the form

$$Q_{i,j}=\sum_{l=0}^{\left[\frac{i}{2}\right]}\sum_{m=0}^{\left[\frac{i}{2}\right]-i}\binom{i-l}{l,2m+\tau_b}\binom{i+\left[\frac{i}{2}\right]-l-m}{\left[\frac{i}{2}\right]-m}\cdot$$
$$\cdot(-a_1)^{\left[\frac{j}{2}\right]-m}\cdot(-a_2)^{i-2l-2m-\tau_b}\cdot(-a_3)^{2m+\tau_b}\cdot \tag{7}$$
$$\cdot(-a_3)^{2m+\tau_b}\cdot(-a_3)^{2m+\tau_b}(-a_4)^j;$$

here $\tau_b=j-2\cdot\left[\frac{j}{2}\right]$.

We presented the recurrent equation (6) for the general form in the form of complete mathematical induction (7), and here we introduced notation such as τ_bb – this is the calculation of the index value by *j* of formula (6).

In the formula (7), $\tau_b=j-2\cdot\left[\frac{j}{2}\right]$, where $\left[\frac{j}{2}\right]$ – entier (*j* mod 2) – taking into account the whole part of the division by 2.

$\binom{a}{b}=\frac{a!}{b!(a-b)!}$ – binomial calculation using the formula [5, 6],

$\binom{i+\left[\frac{i}{2}\right]-l-m}{\left[\frac{i}{2}\right]-m}=\frac{\left(i+\left[\frac{i}{2}\right]-l-m\right)!}{\left(\left[\frac{i}{2}\right]-m\right)!(i-l)!}$ – binomial calculation of the coefficients of the rhaa using the formula,

$\binom{a}{b,c}=\frac{a!}{b!c!(a-b-c)!}$ – the trinomial calculation by the formula [5, 6],

$\binom{i-l}{l,2m+\tau_b}=\frac{(i-l)!}{l!(2m+\tau_b)!(i-2l-2m-\tau_b)!}$ – the trinomial calculation of the coefficients of a series using the formula,

To calculate this induction, we have developed a software procedure in the Maple software environment.

By changing the summation order in (3), we obtain the solution of equation (1) in the form

$$q_r(x,g_r)=\sum_{i=0}^{\infty}x^{(i+r)!}\left(\sum_{j=0}^{\infty}Q_{i,j-1}\right)\frac{\partial^{i-j}}{\partial t^j}g_r(t). \tag{8}$$

Let's write out the first few terms of the series (8) for $r=0, r=1$

$$q_0=g_0-a_2g_0x+\left[-a_1g_{0,t}^{"}-a_3g_{0,t}+\left(a_2^2-a_4\right)g_0\right]\frac{x^2}{2!}+$$
$$+\left[2a_1a_2g_{0,t}^{"}+2a_2a_3g_{0,t}+\left(-a_2^3+2a_2a_4\right)g_0\right]\frac{x^3}{3!}+\ldots$$
$$q_1=g_1-a_2g_1\frac{x^2}{2!}+\left[-a_1g_{1,t}^{"}-a_3g_{1,t}+\left(a_2^2-a_4\right)g_1\right]\frac{x^3}{3!}+$$
$$+\left[2a_1a_2g_{1,t}^{"}+2a_2a_3g_{1,t}+\left(-a_2^3+2a_2a_4\right)g_1\right]\frac{x^4}{4!}+\ldots \tag{9}$$

The partial solution of the inhomogeneous equation (1) in accordance with the recurrent operator method has the form

$$q^*(f)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}Q_{i,j}\frac{\partial^{-(i+j+r)!}}{\partial x^{-(i+j+r)!}}\frac{\partial^j}{\partial t^j}f(x,t) \tag{10}$$

where $Q_{i,j}$ – determined from the condition (2) and (3).

The designation is introduced here

$$\frac{\partial^{-k!}}{\partial x^k}f(x,t)=\underbrace{\int\ldots\int}_{k-times}f(x,t)dx^k$$

The particular solution (9) satisfies zero initial conditions for $x=0$. Thus, the general solution of equation (1) will be

$$q(x,t)=q_0(x,g_0)+q_1(x,g_1)+q^*(f) \tag{11}$$

Let's solve the Cauchy problem for equation (1) with initial conditions

$$\begin{cases} q\,|_{x=0}=\varphi(t) \\ \frac{\partial q}{\partial x}\,|_{x=0}=\psi(t) \end{cases} \tag{12}$$

where $\varphi(t),\psi(t)$ – set functions. Substituting (11) with (8) and (10) into (11) we get

$$\begin{cases} g_0(t)=\varphi(t) \\ \frac{\partial q}{\partial x}|_{x=0}=a_2 g_0(t)+g_0(t)=\psi(t) \end{cases} \tag{13}$$

where $g_1(t)=\psi(t)+a_2\varphi(t)$.

Substituting the found q_0, q_1 into (10), we obtain the solution of the Cauchy problem (1, 2), (11) in the form

$$q(x,t)=(q_0,\varphi(t))+q_1(x,\psi(t))+a_2\varphi(t))+q^*(f)$$
$$=(q_0+a_2q_1)(\varphi(t)+q_1\psi(t))$$

If it is required to satisfy the correctly set initial conditions at t=0 (consistent with the boundary conditions), then equation (1) should be represented as

$$\left(\partial_x^2+a_1\partial_t^2+a_2\partial_x+a_3\partial_t+a_4\right)q^*(t,x)=f^*(t,x) \tag{14}$$

In accordance with the recurrent operator method [7], we are looking for a solution to the homogeneous equation (for $f^*=0$) of equation (10) in the form

$$q_r^*(t,g_r)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}Q_{i,j}^*\frac{t^{(i+j+r)!}}{(i+j+r)!}\frac{\partial^j}{\partial x^j}g_r^*(x),\ r=0,1 \tag{15}$$

where $Q_{i,j}^*$ – constant coefficients determined from condition (4), $g_r^*(x)$– arbitrary differentiable functions determined from initial and boundary conditions. Substituting (15) into (14) we get

$$\begin{aligned}&\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}Q_{i,j}^*\frac{t^{i-2+j+r!}}{(i-2+j+r)!}\frac{\partial^j}{\partial x^j}g_r^*(x)+\\&+a_1\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}Q_{i,j}^*\frac{t^{i+j+r!}}{(i+j+r)!}\frac{\partial^{j+2}}{\partial x^{j+2}}g_r^*(x)+\\&+a_2\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}Q_{i,j}^*\frac{t^{i-1+j+r!}}{(i-1+j+r)!}\frac{\partial^j}{\partial t^j}g_r^*(x)+\\&+a_3\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}Q_{i,j}^*\frac{t^{i+j+r!}}{(i+j+r)!}\frac{\partial^{j+1}}{\partial t^{j+1}}g_r^*(x)+\\&+a_4\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}Q_{i,j}^*\frac{t^{i+j+r!}}{(i+j+r)!}\frac{\partial^j}{\partial x^j}g_r^*(x)=0\end{aligned} \tag{16}$$

By shifting the indices in such a way that the total multiplier $\frac{t^{i-2+j+r!}}{(i-2+j+r)!}\frac{\partial^j}{\partial x^j}g_r^*(x)$ can be taken out of the bracket, we get

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[Q_{i,j}^*+a_1Q_{i,j-2}^*+a_2Q_{i-1,j-1}^*+a_3Q_{i-1,j}^*+a_4Q_{i-2,j}^*\right]\cdot$$
$$\cdot\frac{t^{i-2+j+r!}}{(i-2+j+r)!}\frac{\partial^j}{\partial x^j}g_r^*(x)=0$$

Equating the expression in square brackets to zero, we obtain the following recurrence relation.

$$Q_{i,j}^*=-a_1Q_{i,j-2}^*-a_2Q_{i-1,j-1}^*-a_3Q_{i-1,j}^*-a_4Q_{i-2,j}^* \tag{17}$$

On condition [8, 9]:

$$Q_{i,j}^*=1,\ Q_{i,j}^*=0 \text{ by } i>0 \text{ or } j<0 \tag{18}$$

$Q_{0,0}^*=1$; $Q_{0,1}^*=0$; $Q_{0,2}^*=-a_1$; $Q_{0,3}^*=0$; $Q_{0,4}^*=a_1^2$; $Q_{0,5}^*=0$;

$Q_{1,0}^*=-a_3$; $Q_{1,1}^*=-a_2$; $Q_{1,2}^*=2a_1a_3$; $Q_{1,3}^*=2a_1a_2$;

$Q_{2,0}^*=a_2^2-a_4$; $Q_{2,1}^*=2a_1a_3$; $Q_{2,2}^*=-3a_1a_2^2+2a_1a_4+a_2^2$;

$Q_{3,0}^*=-a_3^2+2a_3a_4$; $Q_{3,1}^*=-3a_1a_3^*+2a_1a_4+a_2^2$.

$$\begin{aligned}Q_{i,j}^*=\sum_{l=0}^{\left[\frac{i}{2}\right]}\sum_{m=0}^{\left[\frac{i}{2}\right]-i}&\binom{i-l}{l,2m+\tau_b}\binom{i+\left[\frac{i}{2}\right]-l-m}{\left[\frac{i}{2}\right]-m}\cdot\\&\cdot(-a_1)^{\left[\frac{j}{2}\right]-m}\cdot(-a_2)^{i-2l-2m-\tau_b}\cdot(-a_3)^{2m+\tau_b}\cdot\\&\cdot(-a_3)^{2m+\tau_b}\cdot(-a_3)^{2m+\tau_b}(-a_4)^j,\end{aligned} \tag{19}$$

here $\tau_b=j-2\cdot\left[\frac{j}{2}\right]$.

By changing the summation order in (17), we obtain the solution of equation (18) in the form

$$q_r^*(x,g_r)=\sum_{i=0}^{\infty}t^{(j+r)!}\left(\sum\nolimits_{j=0}^{i}Q_{i,j-1}^*\right)\frac{\partial^{i-j}}{\partial t^j}g_r(t),\ r=0,1 \tag{20}$$

Let's write out the first few members of the series

$$\begin{aligned}q_0^*=&g_0^*+(-a_3)g_0''''t+\left[-a_1g_{0,x}'''-a_2g_{0,x}''+(a_3^2-a_4)g_0'\right]\frac{t^2}{2!}+\\&+\left[2a_1a_3g_{0,x}'''+2a_2a_3g_{0,x}''+(-a_2^3+2a_2a_4)g_0\right]\frac{t^3}{3!}+\ldots\\q_1^*=&g_0'-a_3g_1\frac{t^2}{2!}+\left[-a_1g_{1,x}''-a_2g_{1,t}''+(a_2^2-a_4)g_1'\right]\frac{t^3}{3!}+\\&+\left[2a_1a_2g_{1,x}''+2a_2a_3g_{1,t}''+(-a_2^3+2a_2a_4)g_{1,x}''\right]\frac{t^4}{4!}\end{aligned} \tag{21}$$

The partial solution of the inhomogeneous equation (12) in accordance with the recurrent operator method [1, 2] has the form

$$q^*(f)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}Q_{i,j}^*\frac{\partial^{-(i+j+2)!}}{\partial x^{-(i+j+2)!}}\frac{\partial^{2j}}{\partial x^{2,j}}f(x,\mathrm{t}) \tag{22}$$

where $Q_{i,j}^*$ – determined from the condition (18) and (4). The designation is introduced here

$$\frac{\partial^{-k!}}{\partial x^k}f(t,x)=\int\underset{k-times}{\ldots}\int f(t,x)dt^k$$

The particular solution (18) satisfies zero initial conditions for $x=0$. Thus, the general solution of equation (12) will be

$$q^*(x,t)=\sum_{r=0}^{1}q_r^*(t,g_r)+q^*(f)$$

Let's solve the Cauchy problem for equation (12) with initial conditions:

$$\begin{cases} q^*(x,t)|_{t=0}=\xi(x) \\ \dfrac{\partial q^*}{\partial t}|_{t=0}=\lambda(x) \end{cases}$$

where from $q_0^*(x)=\xi(x)$, substituting the found functions q_0^*, q_1^* in (20) with the initial conditions (21) in the form

$$q^*=q_0^*(x,\xi(t))+q_1^*(\lambda(t))-a_1\xi(t)+q^*(f) \quad (23)$$

The following data were taken for the computational experiment of the river flow [10]:

$k=D_x=0.18350$, $v=U_x=0.68429$, $\gamma=a=0.991$, $l=7m$, $m=3m$; $s=21m^2$ – the cross-sectional area of the site in question.

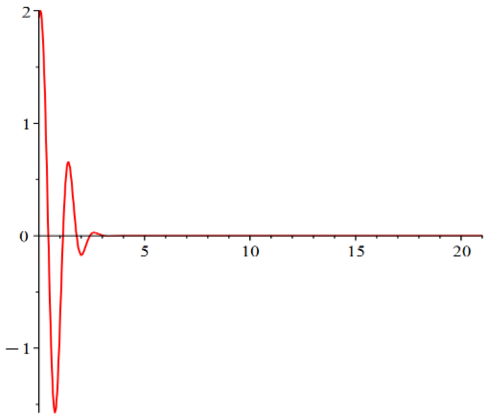

Figure 1. The wave process of the river flow at the initial moment of time at the t=1 s

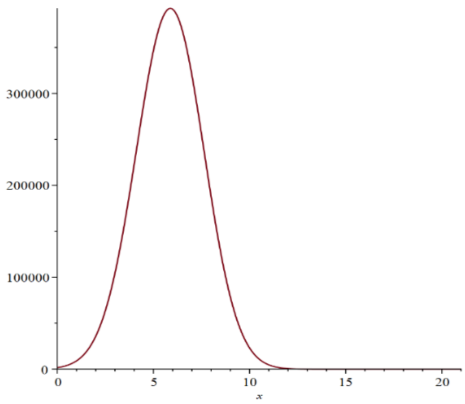

Figure 2. The wave process of flow propagation at the time t=10 s

Also below we will consider the solution of the three-dimensional problem of the wave process, which is a special equation of the Lame equation.

Consider the vector wave equation for a homogeneous isotropic medium called the Lame equation. For a homogeneous isotropic medium in which there are no sources of seismic vibrations, the generalized vector wave equation of Lame is valid.

$$\mu\nabla^2 U+(\lambda+\mu)\,gradU=\rho\frac{\partial^2 U}{\partial t^2} \quad (1')$$

Let us consider a special case of the Lame equation presented in an operator form.

$$\begin{aligned}&(\partial_x^2+\partial_y^2+\partial_z^2-a_1\partial_t^2+\\&+a_2(\partial_x+\partial_y+\partial_z)U(x,y,z,t)=0\end{aligned} \quad (2')$$

A recurrent operator method for a homogeneous isotropic medium is chosen to solve the Lame equation. In accordance with the recurrent operator method, we search for the solution of the equation of the wave process in the form of a series. Where R is the function describing the seismic process, t is the space-time coordinate in time, and x is the time of passage of the wave.

Here $a_1=\frac{\rho}{\mu}$, $a_2=\lambda$. The decision equation (2) we search in the form of a row

$$\begin{aligned}&U_r(x,y,z,g_r)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}U_{i,j,k,p}\cdot\\&\cdot\frac{x^{(i+j+k+r)!}}{(i+j+k+r)!}\cdot\partial_y^j\partial_z^k\partial_t^p g(x,y,z,t);\ r=0,1\end{aligned} \quad (3')$$

here $U_{i,j,k,p}$ – constant coefficients determined from equation (3'), $g(x,y,z,t)$ – arbitrary differentiable functions determined from initial and boundary conditions. Substituting the series (3') into equation (2') in accordance with the recurrent operator, shifting the indices and taking the common multiplier out of the bracket, we have the following recurrent equation

$$\begin{aligned}&\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}\Big[U_{i,j,k,p}+U_{i,j-2,k,p}-U_{i,j,k-2,p}-\\&-a_1U_{i,j,k,p-2}+a_2(U_{i-1,j,k,p}+U_{i-1,j-1,k,p}+U_{i-1,j,k-2,p}\Big]\cdot\\&\cdot\frac{x^{i-2+j+k+r}}{i-2+j+k+r}=0\end{aligned} \quad (4')$$

Equating the expression inside the square bracket to zero, we have a recurrent equation with respect to the series

$$\begin{aligned}&U_{i,j,k,p}=-U_{i,j-2,k,p}-U_{i,j,k-2,p}+a_1U_{i,j,k,p-2}-\\&-a_2U_{i-1,j-1,k,p}-a_2U_{i-1,j,k-1,p}-a_2U_{i-1,j,k,p-1}\end{aligned} \quad (5')$$

We solve this equation under the initial conditions [1, 7]:

$U_{0,0,0,0}=1$, when $i=0$, or $j<0$, or $k<0$, or $p=0$ and $U_{i,j,k,p}=0$, when $i<0$, or $j<0$, or $k<0$, or $p<0$. (6')

By changing the order (4') of summation, we obtain the solution (3') of the equation in the form [11, 12]

$$U_r\left(x,y,z,g_r\right)=\sum_{i=0}^{\infty}x^{(i+j+k+p+r)!}\left(\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}U_{i,i-j,j-k,k-p}\right)\cdot$$
$$\cdot\partial_y^{i-j}\partial_z^{j-k}\partial_t^{k-p}\left(g\left(x,y,z,t\right)\right);\ r=0,1 \qquad (7')$$

Further, performing calculations in the Maple application software package, the following results are obtained. So, by entering the designation of the coefficients, we calculate the equation in the following values

$$\partial_x^k=\frac{\partial^k}{\partial x^k};\ a_1=\frac{\rho}{\mu};$$

$$a_2=\lambda,\ \mu=\lambda=1\ m,\ E=0.5\cdot\frac{10^{10}N}{m^2};$$

$$\omega=2\cdot\pi\cdot\nu=25.12,\ \nu=1Herz,$$

$$\rho=9\cdot\frac{10^{10}N}{m^2};A=1m.$$

For example, consider the process of propagation of seismic waves at a distance of 733 *km* from the city of Almaty, the process occurs in longitude 70°33′10″, latitude 38°46′33″, and depth of 36 *km* 95 *m,* these data are illustrated in Figure 3.

By converting longitude and latitude to spherical coordinates, we obtained the following values for the longitudinal oscillation, that is, denoting for x=1.23137826 *m*., for cross-section for y=0.67630884324 *m.,* for depth for z=36.1038736082·10³ *m.,* this process is considered in a parallelepiped, the length of which is equal to l=7 *m*, the width of the parallelepiped m=3 *m*,

Parallelepiped height h=36 095 m. The volume of the considered section of the seismic zone is 757.995 km³.

The data on the seismic zone are shown in Table 1 ("Satellite maps of the earthquake in Almaty" [13]).

Table 1. Data on the seismic zone of the Almaty region [13]

733 km southwest of Almaty				
Magnitude, points	Depth, km	Latitude	Longitude	Date, time
4.2	36.95	38°45'43"	70°33'10"	12.08.23, 06:29

According to a source from the Institute of Seismology (Figure 4), it was noted that the zones located north (below) Rayymbek Avenue, where there is an increased groundwater level, are considered the most unfavorable from the point of view of seismics in Almaty. The foothill territories of the city in the area of Kok-Tobe Mountain and east of Dostyk Avenue are also unfavorable in terms of seismicity [10].

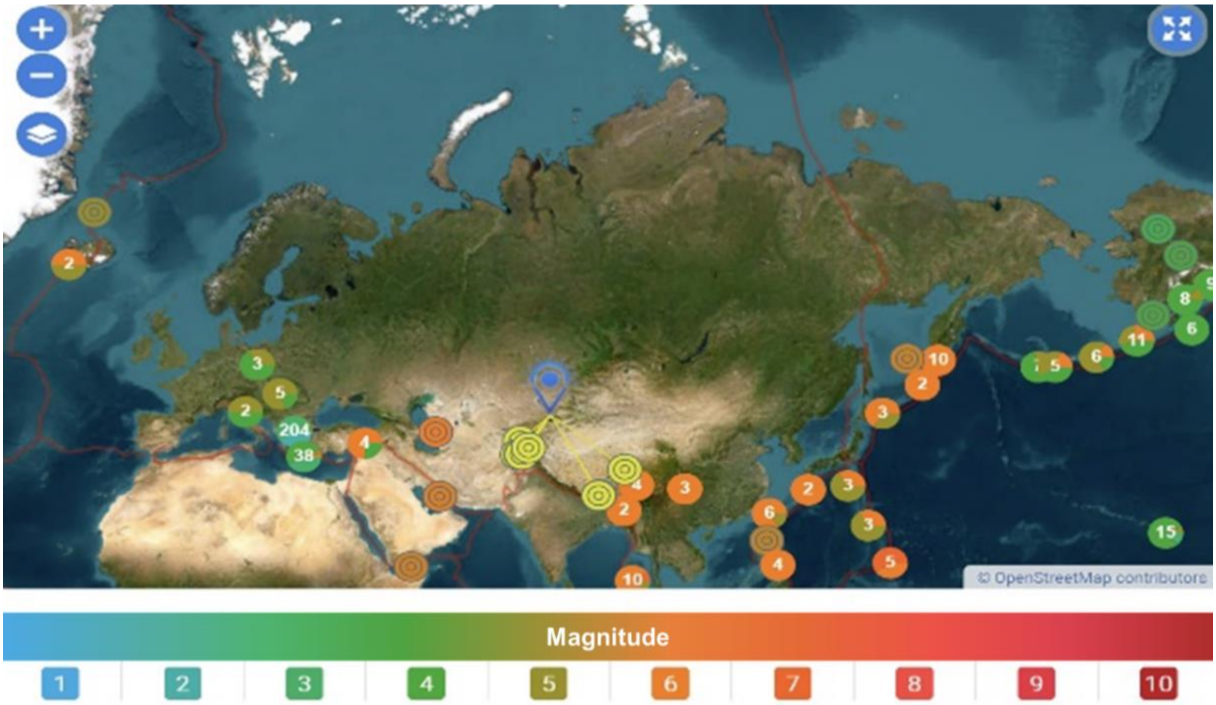

Figure 3. Almaty, 733 km south-west of Almaty. Magnitude 4.2 , 08/12/2023, time 06:29

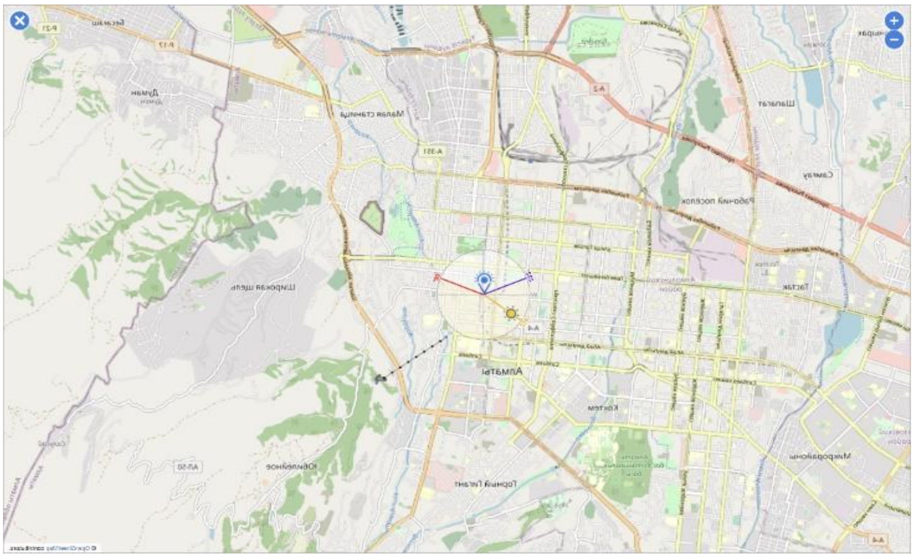

Figure 4. Location of Almaty city in terms of seismic zones

THE RESULTS AND DISCUSSION

Table 2 shows the simulation results obtained in the Maple system, where at the initial moment of time, at t=0.1 *s*, t=1 *s*, at a depth of 36 *km* 95 *m*, it is shown that the process of observing seismic waves is just beginning, Figure 2. Solving the Cauchy problem, the results obtained are shown in this table. Also, taking the values $x=0.0012\cdot10^3$ *m*, $y=0.001\cdot10^3$ *m*, $z=0.003\cdot10^3$ *m* equal, we get the result of modeling the process of seismic wave propagation.

Table 2. Simulation Results of Seismic Wave Propagation

t, s	$x\cdot10^3$, m	$y\cdot10^3$, m	$z\cdot10^3$, m	*u*, m/s²
Study of the process at the initial moment of time				
0.1	0.0012	0.001	0.03	0.9947672305
1	0.007	0.003	0.036095	0.9540476881
Study of a process with boundary conditions along the x-axis				
1	0	0.001	0.003	0.9959709018
20	0	0.0012	0.0184	0.9804607266
30	0	0.003	0.036095	0.9610521864
1	0.00123	0.001	0.00128	0.9964511166
20	0.00326	0.0018	0.018000	0.9929817243
30	0.00700	0.003	0.036095	0.9540472240
Study of a process with boundary conditions along the y-axis				
1	0.00123	0	0.0128	0.9860192323
20	0.00326	0	0.0182	0.9786229332
30	0.00700	0	0.036095	0.9572161210
1	0.00123	0.003	0.012800	0.9807049297
20	0.00326	0.003	0.025600	0.9681164460
30	0.00700	0.003	0.036095	0.9540472240
Study of a process with boundary conditions along the z-axis				
1	0.00123	0.00128	0	0.9974261388
20	0.00326	0.00282	0	0.9935931559
30	0.00700	0.00300	0	0.9894247481
1	0.00123	0.00128	0.036095	0.9617625198
10	0.00326	0.00282	0.036095	0.9580667429
20	0.00700	0.00300	0.036095	0.9540474818
30	0.00700	0.00300	0.036095	0.9540472240

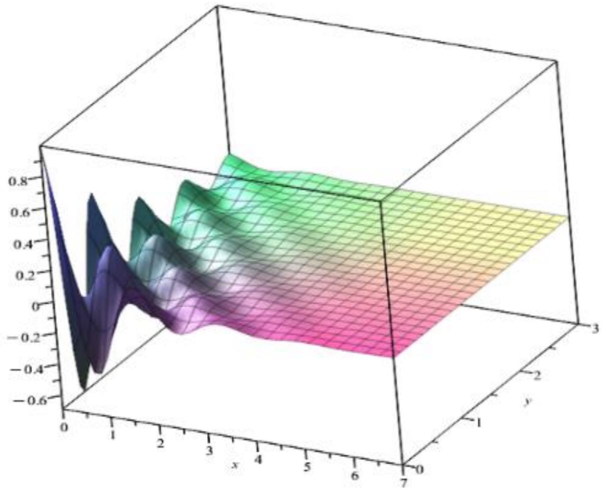

Figure 5. The results of modeling the process of seismic wave propagation at the initial moment of time at t=0.1 s

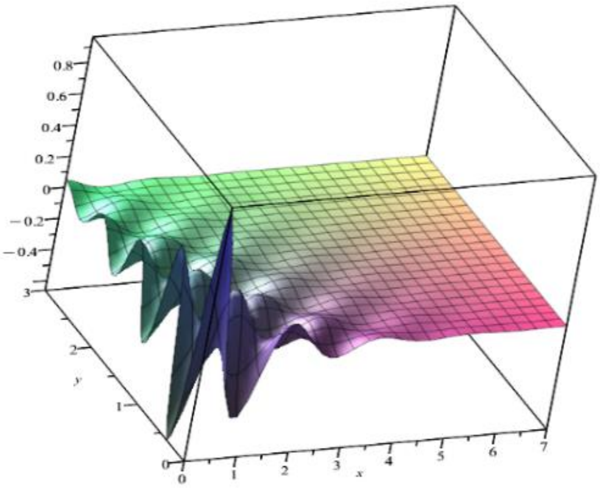

Figure 6. The results of modeling the process of seismic waves at the initial moment of observation time at t=1 s

Table 2 also shows the results of modeling seismic waves under initial and boundary conditions along the *ox* coordinate axis. This process shows that the earth at a depth of z=0.003·10^3 *m*, up to z=0.036095·10^3 meters, at t=1 *s*, in the initial boundary condition on the *ox* axis at x=0, also on the *oy* axis the value of y=0.001 *m*, the earth begins to move away from the elasticity of the earth, that is The process of compression and expansion represents tensor values of the elasticity of the Earth's crust. The result of wave distribution simulation is 0.9959709018 m/s^2. This process was also investigated under the finite boundary condition of x at points x=0.00123·10^3 *m*, x=0.00326·10^3 *m*, x=0.00700·10^3 *m*, respectively, at time values t=1 *s*, t=20 *s*, and also at t=30 *s*, at a depth corresponding to z=0.00128·10^3 *m*, z=0.018000·10^3 *m*, z=0.036095·10^3 *m*, the results are given in the Table 2. The simulation results under the initial boundary conditions are illustrated in Figures 5 and 6. Further, the results of modeling the propagation of seismic waves under initial boundary conditions along the oy coordinate axis are presented below. The results of modeling seismic waves under initial and boundary conditions along the oy coordinate axis are also shown in Table 2. In simulations under initial and boundary conditions along the *oy* coordinate axis, at y=0 and y=0.003·10^3 *m* values, the results obtained are also shown in Table 2. The simulation results obtained along the transverse coordinate of the *oy* axis in the slice show the clearest picture of the wave process than along the *ox* coordinate axis. The simulation results along the *oy* coordinate axis are shown in Figures 5 and 6, where the graph shows the process of wave passage along the *oy* coordinate axis in a slice. The graph here shows the process of wave passage along the transverse coordinate of the axis. According to the transverse coordinate, the directions of wave vibrations of the particle are perpendicular to the direction of wave propagation. The pattern of wave passage is clearly outlined here, and shows how the earth in the transverse coordinate at a depth of 3 and 5 meters also begins to shift along the elasticity of the earth, that is, the process of compression and expansion across gives a clear picture of the passage of the wave.

CONCLUSION

Solutions of the one-dimensional differential equation of the wave process of the river flow are presented. The solution of the equation by this method refers to analytical methods of solution. In many monographs, solutions to this equation are given by numerical methods using difference schemes, while a number of questions arise related to the choice of solution methods, on which the practical feasibility, accuracy and duration of obtaining a solution on a computer depend. In particular: a) an analytical description of a flat or spatial area for which the equation of the wave process and boundary conditions are studied, that is, an analytical description of the coastlines and the river bottom; b) an analytical description of the dependence of the coefficients of the equation on spatial coordinates; c) an analytical description of the dependence on the spatial coordinates of time of the inhomogeneous parts of the equations of the wave process being solved; d) the correct the choice in the difference scheme of the relationship between the spatial steps of the grid, as well as between them and the time sampling step.

The recurrent operator method used in solving problems of the wave process is very effective. Since the solution uses integro-differential operators in the form of a special series with constant coefficients, determined from the recurrent relations associated with the differential equation in question. The particular solutions obtained in this case have a simpler appearance than in other works. With this approach, the general solutions are expressed in terms of arbitrary functions and are not related to the solution of another equation.

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РЕКУРРЕНТ-ОПЕРАТОР ӘДІСІН ҚОЛДАНА ОТЫРЫП, ТОЛҚЫНДЫҚ ПРОЦЕСС ТЕҢДЕУЛЕРІН МОДЕЛЬДЕУ ЖӘНЕ ШЕШУ

П. М. Пирниязова*

Алматы технологиялық университеті, Алматы, Қазақстан

** Байланыс үшін E-mail: pirniyazova1974@gmail.com*

Бұл мақала толқындық процестің бір өлшемді және үш өлшемді мәселесін шешуді қамтамасыз ететін толқындық процеске шолу мақаласы болып табылады. Қарастырылып отырған теңдеу гиперболалық теңдеу болып табылады, оны тек бір өлшемді жағдай үшін алуға болады және өзен ағынының, заттардың тасымалдануының, диффузиялық массаның тасымалдануының және кеуекті орта арқылы ағу процесін сипаттайды. Теңдеудің шешімі қайталану қатынасынан анықталатын тұрақты коэффициенттері бар арнайы қатар түрінде ұсынылған интегралдық-дифференциалдық операторлар түрінде ізделеді. Бұл тәсілмен жалпы шешімдер ерікті функциялар тұрғысынан өрнектеледі және басқа теңдеуді шешумен байланысты емес. Жалпы шешімдердің алынған формасы шекаралық есептерді шешу үшін бастапқы функциялар әдісін қолдануға мүмкіндік береді, өйткені жалпы шешімдерге кіретін ерікті аналитикалық функциялар проблемалық жағдайда көрсетілген бастапқы функциялар тұрғысынан көрсетілуі мүмкін. Жалпы шешімдердің осы формулаларынан аналитикалық функциялардың әр түрлі кластарындағы барлық нақты шешімдерді табу оңай. Әрі қарай, толқындық процесті сипаттайтын Ақсақ теңдеудің ерекше жағдайының шешімі қарастырылады. Ақсақ теңдеудің ерекше жағдайының шешімі қайталанатын оператор әдісін қолдана отырып, қатар түрінде де ізделеді. Толқындық процестің үш өлшемді есебін шешудің алгоритмі бір өлшемді есептің алгоритміне ұқсас. Бұл тәсіл жылу өткізгіштік теориясы мен серпімділік теориясының теңдеулерінің шешімдерін құруда өте тиімді. Алынған нәтижелер графикалық түрде суреттелген.

Түйін сөздер: *толқындық процестің теңдеуі, өзен ағыны, қайталанатын оператор әдісі, интегралдық-дифференциалдық оператор, рекурренттік теңдеу.*

МОДЕЛИРОВАНИЕ И РЕШЕНИЕ УРАВНЕНИЙ ВОЛНОВОГО ПРОЦЕССА С ИСПОЛЬЗОВАНИЕМ РЕКУРРЕНТНО-ОПЕРАТОРНОГО МЕТОДА

П. М. Пирниязова*

Алматинский технологический университет, Алматы, Казахстан

** E-mail для контактов: pirniyazova1974@gmail.com*

Эта обзорная статья, в которой дается решение одномерной и трехмерной задачи о волновом процессе. Рассматриваемое уравнение является гиперболическим уравнением, которое может быть получено только для одномерного случая и описывает процесс речного стока, переноса вещества, диффузионного массопереноса и течения через пористую среду. Решение уравнения ищется в виде интегро-дифференциальных операторов, представленных в виде специального ряда с постоянными коэффициентами, определяемыми из рекуррентного соотношения. При таком подходе общие решения выражаются в терминах произвольных функций и не связаны с решением другого уравнения. Результирующий вид общих решений позволяет применять метод начальных функций для решения краевых задач, поскольку произвольные аналитические функции, входящие в общие решения, могут быть выражены в терминах начальных функций, заданных условием задачи. Из этих формул общих решений легко найти все частные решения в различных классах аналитических функций. Далее, рассматривается решение частного случая уравнения Ламе, описывающего волновой процесс. Решение частного случая уравнения Ламе также ищется в виде ряда с использованием рекуррентно-операторного метода. Алгоритм решения трехмерной задачи о волновом процессе аналогичен алгоритму решения одномерной задачи. Такой подход очень эффективен для построения решений уравнений теории теплопроводности и теории упругости. Полученные результаты проиллюстрированы графически и представлены в таблицах.

Ключевые слова: *волновой процесс, частный случай уравнения Ламе, рекуррентный операторный метод.*